

Gaussian Filtering

Low-pass filtering the resulting grid in the spatial domain (on the sphere) by an averaging Gaussian bell shaped function (which is generated by rotating the Gaussian bell curve) is equivalent to multiplying the spherical harmonic coefficients (i.e. in the frequency domain) with an appropriate Gaussian function depending on the spherical harmonic degree. With the parameter 'filterlength' the width of the gaussian bell can be chosen in the spatial domain measured in meters or in angular units (degrees).

Usually the averaging function in the spatial domain, also called Impulse Response $R(x)$, is written as:

$$R(x) = e^{-1/2 (x/\sigma)^2}$$

It can be shown that the corresponding Transfer Function T_f in the frequency domain is:

$$T_f = e^{-2(\pi\sigma f)^2}$$

Consequently, the Transfer Function T_λ depending on the wavelength $\lambda = 1/f$ is:

$$T_\lambda = e^{-2(\pi\sigma/\lambda)^2}$$

and the Transfer Function depending on the degree n of the spherical harmonics, with $\lambda = 2\pi/n$, is:

$$T_n = e^{-1/2 (\sigma n)^2}$$

The parameter σ defines the width of the Gaussian bell, hence, the filter width (or filter length), which will be denoted here by the symbol Φ , can be set by choosing the value for σ . However, there are different definitions $\Phi = \Phi(\sigma)$ of what is called "filter length", i.e. it is to be defined, between which two points of the Gaussian bell curve the width is measured.

With the parameter 'length_definition', we offer the following 3 definitions for Φ which are commonly in use:

(1) length_definition = "6sigma"

Definition: $\Phi = 6\sigma$

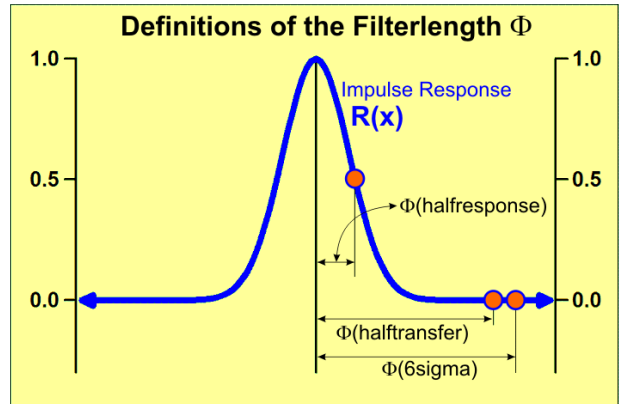
(2) length_definition = "halftransfer"

Definition: $T_\lambda(\lambda=\Phi) = 1/2$

Illustration: the Transfer function T_f (or T_λ) shows how an input pulse is deformed (damped) in the frequency domain. This definition results in a damping factor of $1/2$ for a wavelength of $\lambda = \Phi$ (with a factor of 1 for $\lambda = \infty$)

(3) length_definition = "halfresponse"

Definition: $R(x=\Phi) = 1/2$



The following table summarises the characteristics of the 3 filterlength definitions:

length_definition	$\Phi = \Phi(\sigma)$	$R(x) = e^{-1/2 (x/\sigma)^2}$ ↓ $R(x=\Phi) = ?$	$T_\lambda = e^{-2(\pi\sigma/\lambda)^2}$ ↓ $T_\lambda(\lambda=\Phi) = ?$	$T_n = e^{-1/2 (\sigma n)^2}$ e.g.: $\Phi[\text{rad}] = 5^\circ \cdot (\pi/180^\circ)$ ↓ $T_n(n=?) = 1/2$
6sigma	$\Phi=6\sigma$	$R(x) = e^{-18(x/\Phi)^2}$ ↓ $R(x=\Phi) = 1.523 \cdot 10^{-8}$	$T_\lambda = e^{-(1/18) (\pi\Phi/\lambda)^2}$ ↓ $T_\lambda(\lambda=\Phi) = 0.578$	$T_n = e^{-(1/72) (\Phi n)^2}$ ↓ $T_n(n=81) = 1/2$
halftransfer	$\Phi = \sqrt{(2/\ln 2)}\pi\sigma$ $\Phi = 5.336\sigma$	$R(x) = e^{-(\pi^2/\ln 2) (x/\Phi)^2}$ ↓ $R(x=\Phi) = 6.549 \cdot 10^{-7}$	$T_\lambda = e^{\ln 2 (\Phi/\lambda)^2}$ ↓ $T_\lambda(\lambda=\Phi) = 1/2$	$T_n = e^{-[\ln 2 / (2\pi)^2] (\Phi n)^2}$ ↓ $T_n(n=72) = 1/2$
halfresponse	$\Phi = \sqrt{(2\ln 2)}\sigma$ $\Phi = 1.178\sigma$	$R(x) = e^{-\ln 2 (x/\Phi)^2}$ ↓ $R(x=\Phi) = 1/2$	$T_\lambda = e^{-(\pi^2/\ln 2) (x/\lambda)^2}$ ↓ $T_\lambda(\lambda=\Phi) = 6.549 \cdot 10^{-7}$	$T_n = e^{-[1/(4\ln 2)] (\Phi n)^2}$ ↓ $T_n(n=16) = 1/2$